Chapter 8
The Mundell-Fleming Model

Mundell [108]–Fleming [54] is the IS-LM model adapted to the open economy. Although the framework is rather old and ad hoc the basic framework continues to be used in policy related research (Williamson [132], Hinkle and Montiel [107], MacDonald and Stein [98]). The hallmark of the Mundell-Fleming framework is that goods prices exhibit stickiness whereas asset markets—including the foreign exchange market—are continuously in equilibrium. The actions of policy makers play a major role in these models because the presence of nominal rigidities opens the way for nominal shocks to have real effects. We begin with a simple static version of the model. Next, we present the dynamic but deterministic Mundell-Fleming model due to Dornbusch [39]. Third, we present a stochastic Mundell-Fleming model based on Obstfeld [111].

8.1 A Static Mundell-Fleming Model

This is a Keynesian model where goods prices are fixed for the duration of the analysis. The home country is small in sense that it takes foreign variables as fixed. All variables except the interest rate are in logarithms.

Equilibrium in the goods market is given by an open economy version of the IS curve. There are three determinants of the demand for domestic goods. First, expenditures depend positively on own income $y$ through the absorption channel. An increase in income leads to higher
consumption, most of which is spent on domestically produced goods. Second, domestic goods demand depends negatively on the interest rate $i$ through the investment–saving channel. Since goods prices are fixed, the nominal interest rate is identical to the real interest rate. Higher interest rates reduce investment spending and may encourage a reduction of consumption and an increase in saving. Third, demand for home goods depends positively on the real exchange rate $s + p^* - p$. An increase in the real exchange rate lowers the price of domestic goods relative to foreign goods leading expenditures by residents of the home country as well as residents of the rest of the world to switch toward domestically produced goods. We call this the expenditure switching effect of exchange rate fluctuations. In equilibrium, output equals expenditures which is given by the IS curve

$$y = \delta(s + p^* - p) + \gamma y - \sigma i + g,$$  \hspace{1cm} (8.1)

where $g$ is an exogenous shifter which we interpret as changes in fiscal policy. The parameters $\delta, \gamma,$ and $\sigma$ are defined to be positive with $0 < \gamma < 1$.

As in the monetary model, log real money demand $m^d - p$ depends positively on log income $y$ and negatively on the nominal interest rate $i$ which measures the opportunity cost of holding money. Since the price level is fixed, the nominal interest rate is also the real interest rate, $r$. In logarithms, equilibrium in the money market is represented by the LM curve

$$m - p = \phi y - \lambda i.$$  \hspace{1cm} (8.2)

The country is small and takes the world price level and world interest rate as given. For simplicity, we fix $p^* = 0$. The domestic price level is also fixed so we might as well set $p = 0$.

Capital is perfectly mobile across countries.\footnote{Given the rapid pace at which international financial markets are becoming integrated, analyses under conditions of imperfect capital mobility is becoming less relevant. However, one can easily allow for imperfect capital mobility by modeling both the current account and the capital account and setting the balance of payments to zero (the external balance constraint) as an equilibrium condition. See the end-of-chapter problems.} International capital market equilibrium is given by uncovered interest parity with static
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*Expectations*

\[ i = i^*. \] (8.3)

Substitute (8.3) into (8.1) and (8.2). Totally differentiate the result and rearrange to obtain the two-equation system

\[
dm = \frac{\phi \delta}{1 - \gamma} ds - \left[ \frac{\lambda + \phi \sigma}{1 - \gamma} \right] di^* + \frac{\phi}{1 - \gamma} dg, \] (8.4)

\[
dy = \frac{\delta}{1 - \gamma} ds - \frac{\sigma}{1 - \gamma} di^* + \frac{dg}{1 - \gamma}. \] (8.5)

All of our comparative statics results come from these two equations.

**Adjustment under Fixed Exchange Rates**

*Domestic credit expansion.* Assume that the monetary authorities are credibly committed to fixing the exchange rate. In this environment, the exchange rate is a policy variable. As long as the fix is in effect, we set \( ds = 0 \). Income \( y \) and the money supply \( m \) are endogenous variables.

Suppose the authorities expand the domestic credit component of the money supply. Recall from (1.22) that the monetary base is made up of the sum of domestic credit and international reserves. In the absence of any other shocks \( (di^* = 0, dg = 0) \), we see from (8.4) that there is no long-run change in the money supply \( dm = 0 \) and from (8.5), there is no long-run change in output. The initial attempt to expand the money supply by increasing domestic credit results in an offsetting loss of international reserves. Upon the initial expansion of domestic credit, the money supply does increase. The interest rate must remain fixed at the world rate, however, and domestic residents are unwilling to hold additional money at \( i^* \). They eliminate the excess money by accumulating foreign interest bearing assets and run a temporary balance of payments deficit. The domestic monetary authorities evidently have no control over the money supply in the long run and monetary policy is said to be ineffective as a stabilization tool under a fixed exchange rate regime with perfect capital mobility.

\[ ^{2} \text{Agents expect no change in the exchange rate.} \]
The situation is depicted graphically in Figure 8.1. First, the expansion of domestic credit shifts the LM curve out. To maintain interest parity there is an incipient capital outflow. The central bank defends the exchange rate by selling reserves. This loss of reserves causes the LM curve to shift back to its original position.

Figure 8.1: Domestic credit expansion shifts the LM curve out. The central bank loses reserves to accommodate the resulting capital outflow which shifts the LM curve back in.

Domestic currency devaluation. From (8.4)-(8.5), you have $dy = \frac{\delta}{(1 - \gamma)} ds > 0$ and $dm = \frac{\phi \delta}{(1 - \gamma)} ds > 0$. The expansionary effects of a devaluation are shown in Figure 8.2. The devaluation makes domestic goods more competitive and expenditures switch towards domestic goods. This has a direct effect on aggregate expenditures. In a closed economy, the expansion would lead to an increase in the interest rate but in the open economy under perfect capital mobility, the expansion generates a capital inflow. To maintain the new exchange rate, the central bank accommodates the capital flows by accumulating foreign exchange reserves with the result that the LM curve shifts out.

One feature that the model misses is that in real world economies, the country’s foreign debt is typically denominated in the foreign currency so the devaluation increases the country’s real foreign debt burden.
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\[ r \]
\[ \gamma \]
\[ LM \]
\[ IS \]
\[ FF \]
\[ y_0 \]
\[ y_1 \]

Figure 8.2: Devaluation shifts the IS curve out. The central bank accumulates reserves to accommodate the resulting capital inflow which shifts the LM curve out.

**Fiscal policy shocks.** The results of an increase in government spending are \( dy = \left[ \frac{1}{1-\gamma} \right] dg \) and \( dm = \left[ \frac{\phi}{1-\gamma} \right] dg \) which is expansionary. \( \Leftarrow (137) \) The increase in \( g \) shifts the IS curve to the right and has a direct effect on expenditures. Fiscal policy works the same way as a devaluation and is said to be an effective stabilization tool under fixed exchange rates and perfect capital mobility.

**Foreign interest rate shocks.** An increase in the foreign interest rate has a contractionary effect on domestic output and the money supply, \( dy = -\left( \frac{\sigma}{1-\gamma} \right) di^* \), and \( dm = -\left( \lambda + \phi \sigma / (1-\gamma) \right) di^* \). The increase \( i^* \) creates an incipient capital outflow. To defend the exchange rate, the monetary authorities sell foreign reserves which causes the money supply to contract. The situation is depicted graphically in Figure 8.3.

**Implied International transmissions.** Although we are working with the small-country version of the model, we can qualitatively deduce how policy shocks would be transmitted internationally in a two-country model. If the increase in \( i^* \) was the result of monetary tightening in the large foreign country, output also contracts abroad. We say that
monetary shocks are *positively transmitted* internationally as they lead to positive output comovements at home and abroad. If the increase in \( i^* \) was the result of expansionary foreign government spending, foreign output expands whereas domestic output contracts. Aggregate expenditure shocks are said to be *negatively transmitted* internationally under a fixed exchange rate regime.

A currency devaluation has negative transmission effects. The devaluation of the home currency is equivalent to a revaluation of the foreign currency. Since the domestic currency devaluation has an expansionary effect on the home country, it must have a contractionary effect on the foreign country. A devaluation that expands the home country at the expense of the foreign country is referred to as a *beggar-thy-neighbor policy*.

**Flexible Exchange Rates**

When the authorities do not intervene in the foreign exchange market, \( s \) and \( y \) are endogenous in the system (8.4)-(8.5) and the authorities regain control over \( m \), which is treated as exogenous.

*Domestic credit expansion.* An expansionary monetary policy generates an incipient capital outflow which leads to a depreciation of the
home currency $ds = [(1 - \gamma)/\phi\delta]dm > 0$. The expenditure switching effect of the depreciation increases expenditures on the home good and has an expansionary effect on output $dy = (1/\phi)dm > 0$.

The situation is represented graphically in Figure 8.4 where the expansion of domestic credit shifts the LM curve to the right. In the closed economy, the home interest rate would fall but in the small open economy with perfect capital mobility, the result is an incipient capital outflow which causes the home currency to depreciate ($s$ increases) and the IS curve to shift to the right. The effectiveness of monetary policy is restored under flexible exchange rates.

**Fiscal policy.** Fiscal policy becomes ineffective as a stabilization tool under flexible exchange rates and perfect capital mobility. The situation is depicted in Figure 8.5. An expansion of government spending is represented by an initial outward shift in the IS curve which leads to an incipient capital inflow and an appreciation of the home currency $ds = -(1/\delta)dg < 0$. The resulting expenditure switch forces a subsequent inward shift of the IS curve. The contractionary effects of the induced appreciation offsets the expansionary effect of the government spending leaving output unchanged $dy = 0$. The model predicts an international
version of crowding out. Recipients of government spending expand at the expense of the traded goods sector.

*Interest rate shocks.* An increase in the foreign interest rate leads to an incipient capital outflow and a depreciation given by \( ds = \left[ (\lambda(1 - \gamma) + \sigma \phi) / \phi \delta \right] di^* > 0 \). The expenditure-switching effect of the depreciation causes the IS curve in Figure 8.6 to shift out. The expansionary effect of the depreciation more than offsets the contractionary effect of the higher interest rate resulting in an expansion of output \( dy = (\lambda / \phi) di^* > 0 \).

*International transmission effects.* If the interest rate shock was caused by a contraction in foreign money, the expansion of domestic output would be associated with a contraction of foreign output and monetary policy shocks are negatively transmitted from one country to another under flexible exchange rates. Government spending, on the other hand is positively transmitted. If the increase in the foreign interest rate was precipitated by an expansion of foreign government spending, we would observe expansion in output both abroad and at home.
8.2. DORNBUSCH’S DYNAMIC MUNDELL–FLEMING MODEL

As we saw in Chapter 3, the exchange rate in a free float behaves much like stock prices. In particular, it exhibits more volatility than macroeconomic fundamentals such as the money supply and real GDP. Dornbusch [39] presents a dynamic version of the Mundell–Fleming model that explains excess exchange rate volatility in a deterministic perfect foresight setting. The key feature of the model is that the asset market adjusts to shocks instantaneously while goods market adjustment takes time.

The money market is continuously in equilibrium which is represented by the LM curve, restated here as

\[ m - p = \phi y - \lambda i. \]  \hspace{1cm} (8.6)

To allow for possible disequilibrium in the goods market, let \( y \) denote actual output which is assumed to be fixed, and \( y^d \) denote the demand for home output. The demand for domestic goods depends on the real
exchange rate $s + p^* - p$, real income $y$, and the interest rate $i^3$

$$y^d = \delta(s - p) + \gamma y - \sigma i + g,$$

(8.7)

where we have set $p^* = 0$.

Denote the time derivative of a function $x$ of time with a “dot” $\dot{x}(t) = dx(t)/dt$. Price level dynamics are governed by the rule

$$\dot{p} = \pi(y^d - y),$$

(8.8)

where the parameter $0 < \pi < \infty$ indexes the speed of goods market adjustment.\(^4\) (8.8) says that the rate of inflation is proportional to excess demand for goods. Because excess demand is always finite, the rate of change in goods prices is always finite so there are no jumps in price level. If the price level cannot jump, then at any point in time it is instantaneously fixed. The adjustment of the price-level towards its long-run value must occur over time and it is in this sense that goods prices are sticky in the Dornbusch model.

International capital market equilibrium is given by the uncovered interest parity condition

$$i = i^* + \dot{s}^e,$$

(8.9)

where $\dot{s}^e$ is the expected instantaneous depreciation rate. Let $\bar{s}$ be the steady-state nominal exchange rate. The model is completed by specifying the forward-looking expectations

$$\dot{s}^e = \theta(\bar{s} - s).$$

(8.10)

Market participants believe that the instantaneous depreciation is proportional to the gap between the current exchange rate and its long-run value but to be model consistent, agents must have perfect foresight. This means that the factor of proportionality $\theta$ must be chosen to be consistent with values of the other parameters of the model. This perfect foresight value of $\theta$ can be solved for directly, (as in the chapter

\(^3\)Making demand depend on the real interest rate results in the same qualitative conclusions, but messier algebra.

\(^4\)Low values of $\pi$ indicate slow adjustment. Letting $\pi \to \infty$ allows goods prices to adjust instantaneously which allows the goods market to be in continuous equilibrium.
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appendix) or by the method of undetermined coefficients.\(^5\) Since we can understand most of the interesting predictions of the model without explicitly solving for the equilibrium, we will do so and simply assume that we have available the model consistent value of \(\theta\) such that

\[
\bar{s}^e = \bar{s}. \tag{8.11}
\]

**Steady-State Equilibrium**

Let an ‘overbar’ denote the steady-state value of a variable. The model is characterized by a fixed steady state with \(\bar{s} = \bar{p} = 0\) and

\[
\begin{align*}
\bar{i} &= \bar{i}^*, \\
\bar{p} &= m - \phi y + \lambda \bar{i}, \\
\bar{s} &= \bar{p} + \frac{1}{\delta}[(1 - \gamma) y + \sigma \bar{i} - g].
\end{align*} \tag{8.12, 8.13, 8.14}
\]

Differentiating these long-run values with respect to \(m\) yields \(d\bar{p}/dm = 1\), and \(d\bar{s}/dm = 1\). The model exhibits the sensible characteristic that money is neutral in the long run. Differentiating the long-run values with respect to \(g\) yields \(d\bar{s}/dg = -1/\delta = d(\bar{s} - \bar{p})/dg\). Nominal exchange rate adjustments in response to aggregate expenditure shocks are entirely real in the long run and PPP does not hold if there are permanent shocks to the composition of aggregate expenditures, even in the long run.

**Exchange rate dynamics**

The hallmark of this model is the interesting exchange rate dynamics that follow an unanticipated monetary expansion.\(^6\) Totally differentiating (8.6) but note that \(p\) is instantaneously fixed and \(y\) is always fixed,

\(\theta = \frac{1}{2}[\pi(\delta + \sigma/\lambda) + \sqrt{\pi^2(\delta + \sigma/\lambda)^2 + 4\pi\delta/\lambda}]\).

\(\text{\textsuperscript{6}}\text{This often used experiment brings up an uncomfortable question. If agents have perfect foresight, how a shock be unanticipated?}\)

\(^5\text{The perfect-foresight solution is}\)
the monetary expansion produces a liquidity effect

\[ di = \frac{1}{\lambda} dm < 0. \]  

(8.15)

Differentiate (8.9) while holding \( i^* \) constant and use \( d\bar{s} = dm \) to get \( di = \theta(dm - ds) \). Use this expression to eliminate \( di \) in (8.15). Solving for the instantaneous depreciation yields

\[ ds = \left(1 + \frac{1}{\lambda \theta}\right) dm > d\bar{s}. \]  

(8.16)

This is the famous overshooting result. Upon impact, the instantaneous depreciation exceeds the long-run depreciation so the exchange rate overshoots its long-run value. During the transition to the long run, \( i < i^* \) so by (8.11), people expect the home currency to appreciate. Given that there is a long-run depreciation, the only way that people can rationally expect this to occur is for the exchange rate to initially overshoot the long-run level so that it declines during the adjustment period. This result is significant because the model predicts that the exchange rate is more volatile than the underlying economic fundamen-
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...tals even when agents have perfect foresight. The implied dynamics are illustrated in Figure 8.7.

If there were instantaneous adjustment (τ = ∞), we would immediately go to the long run and would continuously be in equilibrium. So long as τ < ∞, the goods market spends some time in disequilibrium and the economy-wide adjustment to the long-run equilibrium occurs gradually. The transition paths, which we did not solve for explicitly but is treated in the chapter appendix, describe the disequilibrium dynamics. It is in comparison to the flexible-price (long-run) equilibrium that the transitional values are viewed to be in disequilibrium.

There is no overshooting nor associated excess volatility in response to fiscal policy shocks. You are invited to explore this further in the end-of-chapter problems.

8.3 A Stochastic Mundell–Fleming Model

Let’s extend the Mundell-Fleming model to a stochastic environment following Obstfeld [111]. Let \( y_d^t \) be aggregate demand, \( s_t \) be the nominal exchange rate, \( p_t \) be the domestic price level, \( i_t \) be the domestic nominal interest rate, \( m_t \) be the nominal money stock, and \( E_t(\lambda) \) be the mathematical expectation of the random variable \( X_t \) conditioned on date-\( t \) information. All variables except interest rates are in natural logarithms. Foreign variables are taken as given so without loss of generality we set \( p^* = 0 \) and \( i^* = 0 \).

The IS curve in the stochastic Mundell-Fleming model is

\[
y_d^t = \eta(s_t - p_t) - \sigma[i_t - E_t(p_{t+1} - p_t)] + d_t, \tag{8.17}
\]

where \( d_t \) is an aggregate demand shock and \( i_t - E_t(p_{t+1} - p_t) \) is the ex ante real interest rate. The LM curve is

\[
m_t - p_t = y_d^d - \lambda i_t, \tag{8.18}
\]

where the income elasticity of money demand is assumed to be 1. Capital market equilibrium is given by uncovered interest parity

\[
i_t - i^* = E_t(s_{t+1} - s_t). \tag{8.19}
\]
The long-run or the steady-state is not conveniently characterized in a stochastic environment because the economy is constantly being hit by shocks to the non-stationary exogenous state variables. Instead of a long-run equilibrium, we will work with an equilibrium concept given by the solution formed under hypothetically fully flexible prices. Then as long as there is some degree of price-level stickiness that prevents complete instantaneous adjustment, the disequilibrium can be characterized by the gap between sticky-price solution and the shadow flexible-price equilibrium.

Let the shadow values associated with the flexible-price equilibrium be denoted with a ‘tilde.’ The predetermined part of the price level is E$_{t-1}$$\tilde{p}_t$ which is a function of time $t-1$ information. Let $\theta(\tilde{p}_t - E_{t-1}\tilde{p}_t)$ represent the extent to which the actual price level $p_t$ responds at date $t$ to new information where $\theta$ is an adjustment coefficient. The sticky-price adjustment rule is

$$p_t = E_{t-1}\tilde{p}_t + \theta(\tilde{p}_t - E_{t-1}\tilde{p}_t). \quad (8.20)$$

According to this rule, goods prices display rigidity for at most one period. Prices are instantaneously perfectly flexible if $\theta = 1$ and they are completely fixed one-period in advance if $\theta = 0$. Intermediate degrees of price fixity are characterized by $0 < \theta < 1$ which allow the price level at $t$ to partially adjust from its one-period-in-advance predetermined value $E_{t-1}(\tilde{p}_t)$ in response to period $t$ news, $\tilde{p}_t - E_{t-1}\tilde{p}_t$.

The exogenous state variables are output, money, and the aggregate demand shock and they are governed by unit root processes. Output and the money supply are driven by the driftless random walks

$$y_t = y_{t-1} + z_t, \quad (8.21)$$
$$m_t = m_{t-1} + v_t, \quad (8.22)$$

where $z_t \ iid \ N(0, \sigma_z^2)$ and $v_t \ iid \ N(0, \sigma_v^2)$. The demand shock $d_t$ also is a unit-root process

$$d_t = d_{t-1} + \delta_t - \gamma\delta_{t-1}, \quad (8.23)$$

where $\delta_t \ iid \ N(0, \sigma_\delta^2)$. Demand shocks are permanent, as represented by $d_{t-1}$ but also display transitory dynamics where some portion $0 < \gamma < 1$
of any shock $\delta_t$ is reversed in the next period. To solve the model, the first thing you need is to get the shadow flexible-price solution.

**Flexible Price Solution**

Under fully-flexible prices, $\theta = 1$ and the goods market is continuously in equilibrium $y_t = y_t^d$. Let $q_t = s_t - p_t$ be the real exchange rate. Substitute (8.19) into the IS curve (8.17), and re-arrange to get

\[
\tilde{q}_t = \frac{y_t - d_t}{\eta + \sigma} + \left( \frac{\sigma}{\eta + \sigma} \right) E_t \tilde{q}_{t+1}.
\] (8.24)

This is a stochastic difference equation in $\tilde{q}$. It follows that the solution for the flexible-price equilibrium real exchange rate is given by the present value formula which you can get by iterating forward on (8.24). But we won’t do that here. Instead, we will use the method of undetermined coefficients. We begin by conjecturing a guess solution in which $\tilde{q}$ depends linearly on the available date $t$ information

\[
\tilde{q}_t = a_1 y_t + a_2 m_t + a_3 d_t + a_4 \delta_t.
\] (8.25)

We then deduce conditions on the $a-$coefficients such that (8.25) solves the model. Since $m_t$ does not appear explicitly in (8.24), it probably is the case that $a_2 = 0$. To see if this is correct, take time $t$ conditional expectations on both sides of (8.25) to get

\[
E_t \tilde{q}_{t+1} = a_1 y_t + a_2 m_t + a_3 (d_t - \gamma \delta_t).
\] (8.26)

Substitute (8.25) and (8.26) into (8.24) to get

\[
\Rightarrow (139)
\]

\[
a_1 y_t + a_2 m_t + a_3 d_t + a_4 \delta_t = \frac{y_t - d_t}{\eta + \sigma} + \sigma \frac{[a_1 y_t + a_2 m_t + a_3 (d_t - \gamma \delta_t)]}{\eta + \sigma}.
\]

\[\text{Recursive backward substitution in (8.23) gives, } d_t = \delta_t + (1 - \gamma) \delta_{t-1} + (1 - \gamma) \delta_{t-2} + \cdots. \] Thus the demand shock is a quasi-random walk without drift in that a shock $\delta_t$ has a permanent effect on $d_t$, but the effect on future values $(1 - \gamma)$ is smaller than the current effect.
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Now equate the coefficients on the variables to get

\[ a_1 = \frac{1}{\eta} = -a_3, \]
\[ a_2 = 0, \]
\[ a_4 = \frac{\gamma}{\eta} \left( \frac{\sigma}{\eta + \sigma} \right). \]

The flexible-price solution for the real exchange rate is

\[ \tilde{q}_t = \frac{y_t - d_t}{\eta} + \frac{\gamma}{\eta} \left( \frac{\sigma}{\eta + \sigma} \right) \delta_t, \tag{8.27} \]

where indeed nominal (monetary) shocks have no effect on \( \tilde{q}_t \). The real exchange rate is driven only by real factors—supply and demand shocks.

Since both of these shocks were assumed to evolve according to unit root process, there is a presumption that \( \tilde{q}_t \) also is a unit root process. A permanent shock to supply \( y_t \) leads to a real depreciation. Since \( \gamma\sigma / (\eta(\eta + \sigma)) < (1/\eta) \), a permanent shock to demand \( \delta_t \) leads to a real appreciation.\(^8\)

To get the shadow price level, start from (8.18) and (8.19) to get \( \tilde{p}_t = m_t - y_t + \lambda E_t(s_{t+1} - s_t) \). If you add \( \lambda \tilde{p}_t \) to both sides, add and subtract \( \lambda E_t \tilde{p}_{t+1} \) to the right side and rearrange, you get

\[ (1 + \lambda) \tilde{p}_t = m_t - y_t + \lambda E_t(\tilde{q}_{t+1} - \tilde{q}_t) + \lambda E_t \tilde{p}_{t+1}. \tag{8.28} \]

By (8.27), \( E_t(\tilde{q}_{t+1} - \tilde{q}_t) = [\gamma/(\eta + \sigma)] \delta_t \), which you can substitute back into (8.28) to obtain the stochastic difference equation

\[ \tilde{p}_t = \frac{m_t - y_t}{1 + \lambda} + \frac{\lambda \gamma}{(\eta + \sigma)(1 + \lambda)} \delta_t + \frac{\lambda}{1 + \lambda} E_t \tilde{p}_{t+1}. \tag{8.29} \]

Now solve (8.29) by the MUC. Let

\[ \tilde{p}_t = b_1 m_t + b_2 y_t + b_3 d_t + b_4 \delta_t, \tag{8.30} \]

be the guess solution. Taking expectations conditional on time-\( t \) information gives

\[ E_t \tilde{p}_{t+1} = b_1 m_t + b_2 y_t + b_3 (d_t - \gamma \delta_t). \tag{8.31} \]

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\(^8\)Here is another way to motivate the null hypothesis that the real exchange rate follows a unit root process in tests of long-run PPP covered in Chapter 7.
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Substitute (8.31) and (8.30) into (8.29) to get

\[
\begin{aligned}
  b_1 m_t &+ b_2 y_t + b_3 d_t + b_4 \delta_t \\
  &= \frac{m_t - y_t}{1 + \lambda} + \frac{\lambda \gamma}{(1 + \lambda)(\eta + \sigma)} \delta_t \\
  &+ \frac{\lambda}{1 + \lambda} [b_1 m_t + b_2 y_t + b_3 (d_t - \gamma \delta_t)].
\end{aligned}
\]  

(8.32)

Equate coefficients on the variables to get

\[
\begin{aligned}
  b_1 &= 1 = -b_2, \\
  b_3 &= 0, \\
  b_4 &= \frac{\lambda \gamma}{(1 + \lambda)(\eta + \sigma)}. \\
\end{aligned}
\]  

(8.33)

Write the flexible-price equilibrium solution for the price level as

\[
\tilde{p}_t = m_t - y_t + \alpha \delta_t,
\]  

(8.34)

where

\[
\alpha = \frac{\lambda \gamma}{(1 + \lambda)(\eta + \sigma)}. 
\]

A supply shock \( y_t \) generates shadow deflationary pressure whereas demand shocks \( \delta_t \) and money shocks \( m_t \) generate shadow inflationary pressure.

The shadow nominal exchange rate can now be obtained by adding \( \tilde{q}_t + \tilde{p}_t \)

\[
\tilde{s}_t = m_t + \left( \frac{1 - \eta}{\eta} \right) y_t - d_t + \left( \frac{\gamma \sigma}{\eta (\eta + \sigma)} + \alpha \right) \delta_t.
\]  

(8.35)

Positive monetary shocks unambiguously lead to a nominal depreciation but the effect of a supply shock on the shadow nominal exchange rate depends on the magnitude of the expenditure switching elasticity, \( \eta \). You are invited to verify that a positive demand shock \( \delta_t \) lowers the nominal exchange rate.
Collecting the equations that form the flexible-price solution we have

\[ y_t = y_{t-1} + z_t = y(z_t), \]
\[ \ddot{q}_t = \frac{y_t - d_t}{\eta} + \frac{\gamma \sigma}{\eta (\eta + \sigma)} \delta_t = \ddot{q}(z_t, \delta_t), \]
\[ \ddot{p}_t = m_t - y_t + \alpha \delta_t = \ddot{p}(z_t, \delta_t, v_t). \]

The system displays a triangular structure in the exogenous shocks. Only supply shocks affect output, demand and supply shocks affect the real exchange rate, while supply, demand, and monetary shocks affect the price level. We will revisit the implications of this triangular structure in Chapter 8.4.

**Disequilibrium Dynamics**

To obtain the sticky-price solution with \( 0 < \theta < 1 \), substitute the solution (8.34) for \( \ddot{p}_t \) into the price adjustment rule (8.20), to get

\[ p_t = m_{t-1} - y_{t-1} + \theta [v_t - z_t + \alpha \delta_t]. \]

Next, add and subtract \( (v_t - z_t + \alpha \delta_t) \) to the right side and rearrange to get

\[ p_t = \ddot{p}_t - (1 - \theta)[v_t - z_t + \alpha \delta_t]. \quad (8.36) \]

The gap between \( p_t \) and \( \ddot{p}_t \) is proportional to current information \( (v_t - z_t + \alpha \delta_t) \), which we’ll call news. You will see below that the gap between all disequilibrium values and their shadow values are proportional to this news variable. Monetary shocks \( v_t \) and demand shocks \( \delta_t \) cause the price level to lie below its equilibrium value \( \ddot{p}_t \) while supply shocks \( z_t \) cause the current price level to lie above its equilibrium value.\(^{9}\) Since the solution for \( p_t \) does not depend on lagged values of the shocks, the deviation from full-price flexibility values generated by current period shocks last for only one period.

Next, solve for the real exchange rate. Substitute (8.36) and aggregate demand from the IS curve (8.17) into the LM curve (8.18) to

\(^{9}\)The price-level responses to the various shocks conform precisely to the predictions from the aggregate-demand, aggregate-supply model as taught in principles of macroeconomics.
8.3. A STOCHASTIC MUNDELL–FLEMING MODEL

get

\[ m_t - \tilde{p}_t + (1 - \theta)[v_t - z_t + \alpha \delta_t] = d_t + \eta q_t - (\sigma + \lambda)(E_t q_{t+1} - q_t) - \lambda E_t(p_{t+1} - p_t). \]  

(8.37)

By (8.36) and (8.34) you know that

\[ E_t(p_{t+1} - p_t) = -\alpha \delta_t + (1 - \theta)[v_t - z_t + \alpha \delta_t]. \]  

(8.38)

Substitute (8.38) and \( \tilde{p}_t \) into (8.37) to get the stochastic difference equation in \( q_t \)

\[ \eta + \sigma + \lambda)q_t = y_t - d_t + (1 - \theta)(1 + \lambda)(v_t - z_t) - \theta(1 + \lambda)\alpha \delta_t + (\sigma + \lambda)E_t q_{t+1}. \]  

(8.39)

Let the conjectured solution be

\[ q_t = c_1 y_t + c_2 d_t + c_3 \delta_t + c_4 v_t + c_5 z_t. \]  

(8.40)

It follows that

\[ E_t q_{t+1} = c_1 y_t + c_2 (d_t - \gamma \delta_t). \]  

(8.41)

Substitute (8.40) and (8.41) into (8.39) to get

\[(\eta + \sigma + \lambda)\left[ c_1 y_t + c_2 d_t + c_3 \delta_t + c_4 v_t + c_5 z_t \right] = y_t - d_t + (1 - \theta)(1 + \lambda)(v_t - z_t) - \theta(1 + \lambda)\alpha \delta_t + (\sigma + \lambda)\left[ c_1 y_t + c_2 (d_t - \gamma \delta_t) \right].\]

Equating coefficients gives

\[
\begin{align*}
c_1 &= \frac{1}{\eta} = -c_2, \\
c_3 &= \frac{\gamma(\sigma + \lambda) - \eta \alpha \theta(1 + \lambda)}{\eta(\eta + \sigma + \lambda)} \\
c_4 &= \frac{(1 - \theta)(1 + \lambda)}{\eta + \sigma + \lambda} = -c_5,
\end{align*}
\]

and the solution is

\[ q_t = \frac{y_t - d_t}{\eta} + \frac{\gamma(\sigma + \lambda) - \eta \alpha \theta(1 + \lambda)}{\eta(\eta + \sigma + \lambda)} d_t + \frac{(1 - \theta)(1 + \lambda)}{\eta + \sigma + \lambda} (v_t - z_t). \]  

(8.41)
CHAPTER 8. THE MUNDELL-FLEMING MODEL

Using the definition of $\alpha$ and (8.27) to eliminate $(y_t - d_t)/\eta$, rewrite the solution in terms of $\tilde{q}_t$ and news

$$q_t = \tilde{q}_t + \frac{(1 + \lambda)(1 - \theta)}{\eta + \sigma + \lambda}[v_t - z_t + \alpha \delta_t]. \quad (8.42)$$

Nominal shocks have an effect on the real exchange rate due to the rigidity in price adjustment. Disequilibrium adjustment in the real exchange rate runs in the opposite direction of price level adjustment. Monetary shocks and demand shocks cause the real exchange rate to temporarily rise above its equilibrium value whereas supply shocks cause the real exchange rate to temporarily fall below its equilibrium value.

To get the nominal exchange rate $s_t = q_t + p_t$, add the solutions for $q_t$ and $p_t$

$$s_t = \tilde{s}_t + (1 - \eta - \sigma)(1 - \theta)[v_t - z_t + \alpha \delta_t]. \quad (8.43)$$

The solution displays a modified form of exchange-rate overshooting under the presumption that $\eta + \sigma < 1$ in that a monetary shock causes the exchange rate to rise above its shadow value $\tilde{s}_t$. In contrast to the Dornbusch model, both nominal and real shocks generate modified exchange-rate overshooting. Positive demand shocks cause $s_t$ to rise above $\tilde{s}_t$ whereas supply shocks cause $s_t$ to fall below $\tilde{s}_t$.

To determine excess goods demand, you know that aggregate demand is

$$y_t^d = \eta q_t - \sigma E_t(\Delta q_{t+1}) + d_t.$$  

(142)$\Rightarrow$

Taking expectations of (8.42) yields

$$E_t(\Delta q_{t+1}) = \frac{\gamma}{\eta + \sigma} \delta_t - \frac{(1 + \lambda)(1 - \theta)}{(\eta + \sigma + \lambda)}[v_t - z_t + \alpha \delta_t].$$

Substitute this and $q_t$ from (8.42) back into aggregate demand and rearrange to get

$$y_t^d = y_t + \frac{(1 + \lambda)(1 - \theta)(\eta + \sigma)}{(\eta + \sigma + \lambda)}[v_t - z_t + \alpha \delta_t]. \quad (8.44)$$

Goods market disequilibrium is proportional to the news $v_t - z_t + \alpha \delta_t$. Monetary shocks have a short-run effect on aggregate demand, which is the stochastic counterpart to the statement that monetary policy is an effective stabilization tool under flexible exchange rates.
8.4 VAR analysis of Mundell–Fleming

Even though it required tons of algebra to solve, the stochastic Mundell–Fleming with one-period nominal rigidity is still too stylized to take seriously in formulating econometric specifications. Modeling lag dynamics in price adjustment is problematic because we don’t have a good theory for how prices adjust or for why they are sticky. Tests of overidentifying restrictions implied by dynamic versions of the Mundell–Fleming model are frequently rejected, but the investigator does not know whether it is the Mundell-Fleming theory that is being rejected or one of the auxiliary assumptions associated with the parametric econometric representation of the theory.\(^{10}\)

Sims [129] views the restrictions imposed by explicitly formulated macroeconometric models to be incredible and proposed the unrestricted VAR method to investigate macroeconomic theory without having to assume very much about the economy. In fact, just about the only thing that you need to assume are which variables to include in the analysis. Unrestricted VAR estimation and accounting methods are described in Chapter 2.1.

The Eichenbaum and Evans VAR

Eichenbaum and Evans [41] employ the Sims VAR method to the five dimensional vector-time-series consisting of i) US industrial production, ii) US CPI, iii) A US monetary policy variable iv) US–foreign nominal interest rate differential, and v) US real exchange rate. They considered two measures of monetary policy. The first was the ratio of the logarithm of nonborrowed reserves to the logarithm of total reserves. The second was the federal funds rate. They estimated separate VARs using exchange rates and interest rates for each of five countries: Japan, Germany, France, Italy, and the UK with monthly observations from 1974.1 through 1990.5.

Here, we will re-estimate the Eichenbaum–Evans VAR and do the associated VAR accounting using monthly observations for the US, UK, Germany, and Japan from 1973.1 to 1998.1. All variables except inter-

\(^{10}\)See Papell [117].
interest rates are in logarithms. Let $y_t$ be US industrial production, $p_t$ be the US consumer price index, $nbr_t$ be the log of non-borrowed bank reserves divided by the log of total bank reserves, $i_t - i^*_t$ be the 3 month US-foreign nominal interest rate differential, $q_t$ be the real exchange rate, and $s_t$ be the nominal exchange rate.\(^{11}\) For each US–foreign country pair, two separate VARs were run—one using the real exchange rate and one with the nominal exchange rate. In the first system, the VAR is estimated for the 5-dimensional vector $\mathbf{x}_t = (y_t, p_t, nbr_t, i_t - i^*_t, q_t)$. In the second system, we used $\mathbf{x}_t = (y_t, p_t, nbr_t, i_t - i^*_t, s_t)$.\(^{12}\)

The first row of plots in Figure 8.8 shows the impulse response of the log real exchange rate for the US-UK, US-Germany, and US-Japan, following a one-standard deviation shock to $nbr_t$. An increase in $nbr_t$ corresponds to a positive monetary shock. The second row shows the responses of the log nominal exchange rate with the same countries to a one-standard deviation shock to $nbr_t$.

Both the real and nominal exchange rates are found to depreciate upon impact but the maximal nominal depreciation occurs some months after the initial shock. The impulse response of both exchange rates is hump-shaped. There is evidently evidence of overshooting, but it is different from Dornbusch overshooting which is instantaneous. This unrestricted VAR response pattern has come to be known as delayed overshooting.

Long-horizon (36 months ahead) forecast-error variance decompositions of nominal exchange rates attributable to orthogonalized monetary shocks are 16 percent for the UK, 24 percent for Germany, and 10 percent for Japan. For real exchange rates, the percent of variance attributable to monetary shocks is 23 percent for the UK and Germany, and 9 percent for Japan. Evidently, nominal shocks are pretty important in driving the dynamics of the real exchange rate.

\(^{11}\)Interest rates for the US and UK are the secondary market 3-month Treasury Bill rate. For Germany, I used the interbank deposit rate. For Japan, the interest rate is the Japanese lending rate from the beginning of the sample to 1981.8, and is the private bill rate from 1981.9 to 1998.1

\(^{12}\)Using BIC (Chapter 2, equation 2.3) with the updated data indicated that the VARs required 3 lags. To conform with Eichenbaum and Evans, I included 6 lags and a linear trend.

Clarida-Gali Structural VAR

In Chapter 2.1, we discussed some potential pitfalls associated with the unrestricted VAR methodology. The main problem is that the unrestricted VAR analyzes a reduced form of a structural model so we do not necessarily learn anything about the effect of policy interventions on the economy. For example, when we examine impulse responses from an innovation in $y_t$, we do not know whether the underlying cause was due to a shock to aggregate demand or to aggregate supply or an expansion of domestic credit.

Blanchard and Quah [15] show how to use economic theory to place identifying restrictions on the VAR, resulting in so-called struc-
tural VARs.\textsuperscript{13} Clarida and Gali \cite{28} employ Blanchard-Quah’ structural VAR method using restrictions implied by the stochastic Mundell-Fleming model. To see how this works, consider the 3-dimensional vector, 
\[ x_t = (\Delta(y_t - y^*_t), \Delta(p_t - p^*_t), \Delta(q_t))^\prime, \]
where \( y \) is log industrial production, \( p \) is the log price level, and \( q \) is the log real exchange rate and starred variables are for the foreign country. Given the processes that govern the exogenous variables (8.21) and (8.22), the stochastic Mundell-Fleming model predicts that income and the real exchange rate are unit root processes, so the VAR should be specified in terms of first-differenced observations. The triangular structure also informs us that the variables are not cointegrated, since each of the variables are driven by a different unit root process.\textsuperscript{14}

As described in Chapter 2.1, first fit a \( p \)-th order VAR for \( x_t \) and get the Wold moving average representation
\[ x_t = \sum_{j=0}^{\infty} (C_j L^j)\xi_t = C(L)\xi_t, \tag{8.45} \]
where \( \text{E}(\xi_\prime \xi_0) = \Sigma \), \( C_0 = I \), and \( C(L) = \sum_{j=0}^{\infty} C_j L^j \) is the one-sided matrix polynomial in the lag operator \( L \). The theory predicts that in the long run, \( x_t \) is driven by the three dimensional vector of aggregate supply, aggregate demand, and monetary shocks, \( v_t = (z_t, \delta_t, v_t^\prime) \).

The economic structure embodied in the stochastic Mundell-Fleming model is represented by
\[ x_t = \sum_{j=0}^{\infty} (F_j L^j)\psi_t = F(L)\psi_t. \tag{8.46} \]
Because the underlying structural innovations are not observable, you are allowed to make one normalization. Take advantage of it by setting \( \text{E}(\psi_\prime \psi_0) = I \). The orthogonality between the various structural shocks is an identifying assumption. To map the innovations \( \xi_t \) from the unrestricted VAR into structural innovations \( \psi_t \), compare (8.45) and (8.46). It follows that
\[ \xi_t = F_0 \psi_t \Rightarrow \xi_{t-j} = F_0 \psi_{t-j} \Rightarrow C_j \xi_{t-j} = C_j F_0 \psi_{t-j} = F_j \psi_{t-j}. \]
\textsuperscript{13}They are only identifying restrictions, however, and cannot be tested.
\textsuperscript{14}Cointegration is discussed in Chapter 2.6.
To summarize

\[ F_j = C_j F_0 \quad \text{for all } j \Rightarrow F(1) = C(1) F_0. \] (8.47)

Given the \( C_j \), which you get from unrestricted VAR accounting, (8.47) says you only need to determine \( F_0 \) after which the remaining \( F_j \) follow.

In our 3-dimensional system, \( F_0 \) is a \( 3 \times 3 \) matrix with 9 unique elements. To identify \( F_0 \), you need 9 pieces of information. Start with,

\[ \Sigma = G' G = E(\varepsilon_t \varepsilon_t') = F_0 E(\varepsilon_t \varepsilon_t') F_0' = F_0 F_0' \] where \( G \) is the unique upper triangular Choleski decomposition of the error covariance matrix \( \Sigma \). To summarize

\[ \Sigma = G' G = F_0 F_0'. \] (8.48)

Let \( g_{ij} \) be the \( ij \)th element of \( G \) and \( f_{ij,0} \) be the \( ij \)th element of \( F_0 \). Writing (8.48) out gives

\[
\begin{align*}
g_{11}^2 &= f_{11,0}^2 + f_{12,0}^2 + f_{13,0}^2, \\
g_{11}g_{12} &= f_{11,0}f_{21,0} + f_{12,0}f_{22,0} + f_{13,0}f_{23,0}, \\
g_{11}g_{13} &= f_{11,0}f_{31,0} + f_{12,0}f_{32,0} + f_{13,0}f_{33,0}, \\
g_{12}^2 + g_{22}^2 &= f_{21,0}^2 + f_{22,0}^2 + f_{23,0}^2, \\
g_{12}g_{13} + g_{22}g_{23} &= f_{21,0}f_{31,0} + f_{22,0}f_{32,0} + f_{23,0}f_{33,0}, \\
g_{13}^2 + g_{23}^2 + g_{33}^2 &= f_{31,0}^2 + f_{32,0}^2 + f_{33,0}^2.
\end{align*}
\] (8.49 - 8.54)

\( G \) has 6 unique elements so this decomposition gives you 6 equations in 9 unknowns. You still need three additional pieces of information. Get them from the long-run predictions of the theory.

Stochastic Mundell-Fleming predicts that neither demand shocks nor monetary shocks have a long-run effect on output which we represent by setting \( f_{12}(1) = 0 \) and \( f_{13}(1) = 0 \), where \( f_{ij}(1) \) is the \( ij \)th element of \( F(1) = \sum_{j=0}^{\infty} F_j \). The model also predicts that money has no long-run effect on the real exchange rate \( f_{33}(1) = 0 \). Since \( F(1) = C(1) F_0 \), impose these three restrictions by setting

\[
\begin{align*}
f_{13}(1) &= 0 = c_{11}(1)f_{13,0} + c_{12}(1)f_{23,0} + c_{13}(1)f_{33,0}, \\
f_{12}(1) &= 0 = c_{11}(1)f_{12,0} + c_{12}(1)f_{22,0} + c_{13}(1)f_{32,0}, \\
f_{33}(1) &= 1 = c_{31}(1)f_{13,0} + c_{32}(1)f_{23,0} + c_{33}(1)f_{33,0}.
\end{align*}
\] (8.55 - 8.57)
(8.49)–(8.57) form a system of 9 equations in 9 unknowns and implicitly define $F_0$. Once the $F_j$ are obtained, you can do impulse response analyses and forecast error variance decompositions using the ‘structural’ response matrices $F_j$.

Table 8.1: Structural VAR forecast error variance decompositions for real exchange rate depreciation

<table>
<thead>
<tr>
<th></th>
<th>Supply</th>
<th>Demand</th>
<th>Money</th>
<th></th>
<th>Supply</th>
<th>Demand</th>
<th>Money</th>
</tr>
</thead>
<tbody>
<tr>
<td>Britain</td>
<td>0.378</td>
<td>0.240</td>
<td>0.382</td>
<td>1 month</td>
<td>0.331</td>
<td>0.211</td>
<td>0.458</td>
</tr>
<tr>
<td>Germany</td>
<td>0.016</td>
<td>0.234</td>
<td>0.750</td>
<td></td>
<td>0.066</td>
<td>0.099</td>
<td>0.835</td>
</tr>
<tr>
<td>Japan</td>
<td>0.872</td>
<td>0.011</td>
<td>0.117</td>
<td></td>
<td>0.810</td>
<td>0.071</td>
<td>0.119</td>
</tr>
</tbody>
</table>

Clarida and Gali estimate a structural VAR using quarterly data from 1973.3 to 1992.4 for the US, Germany, Japan, and Canada. Their impulse response analysis revealed that following a one-standard deviation nominal shock, the real exchange rate displayed a hump shape, initially depreciating then subsequently appreciating. Real exchange rate dynamics were found to display delayed overshooting.

We’ll re-estimate the structural VAR using 4 lags and monthly data for the US, UK, Germany, and Japan from 1976.1 through 1997.4. The structural impulse response dynamics of the levels of the variables are displayed in Figure 8.9. As predicted by the theory, supply shocks lead to a permanent real deprecation and demand shocks lead to a permanent real appreciation. The US-UK real exchange rate does not exhibit delayed overshooting in response to monetary shocks. The real dollar-pound rate initially appreciates then subsequently depreciates following a positive monetary shock. The real dollar-deutschemark rate displays overshooting by first depreciating and then subsequently appreciating. The real dollar-yen displays Dornbusch-style overshooting. Money shocks are found to contribute a large fraction of the forecast error variance both the long run as well as at the short run for the real exchange rate. The decompositions at the 1-month and 36-month forecast horizons are reported in Table 8.1.
8.4. VAR ANALYSIS OF MUNDELL–FLEMING

CHAPTER 8. THE MUNDELL-FLEMING MODEL

Mundell-Fleming Models Summary

1. The hallmark of Mundell-Fleming models is that they assume that goods prices are sticky. Many people think of Mundell–Fleming models synonymously with sticky-price models. Because there exist nominal rigidities, these models invite an assessment of monetary (and fiscal) policy interventions under both fixed and flexible exchange rates. The models also provide predictions regarding the international transmission of domestic shocks and co-movements of macroeconomic variables at home and abroad.

2. The Dornbusch version of the model exploits the slow adjustment in the goods market combined with the instantaneous adjustment in the asset markets to explain why the exchange rate, which is the relative price of two monies (assets), may exhibit more volatility than the fundamentals in a deterministic and perfect foresight environment. Explaining the excess volatility of the exchange rate is a recurring theme in international macroeconomics.

3. The dynamic stochastic version of the model is amenable to empirical analysis. The model provides a useful guide for doing unrestricted and structural VAR analysis.
Appendix: Solving the Dornbusch Model

From (8.9) and (8.11), we see that the behavior of \( i(t) \) is completely determined by that of \( s(t) \). This means that we need only determine the differential equations governing the exchange rate and the price level to obtain a complete characterization of the system’s dynamics.

Substitute (8.9) and (8.11) into (8.6). Make use of (8.13) and rearrange to obtain

\[
\dot{s}(t) = \frac{1}{\lambda}[p(t) - \bar{p}]. \tag{8.58}
\]

To obtain the differential equation for the price level, begin by substituting (8.58) into (8.9), and then substituting the result into (8.8) to get

\[
\dot{p}(t) = \pi[\delta(s(t) - p(t)) + (\gamma - 1)y - \sigma\bar{r} - \frac{\sigma}{\lambda}(p(t) - \bar{p}) + g]. \tag{8.59}
\]

However, in the long run

\[
0 = \pi[\delta(\bar{s} - \bar{p}) + (\gamma - 1)y - \sigma\bar{r} + g], \tag{8.60}
\]

the price dynamics are more conveniently characterized by

\[
\dot{p}(t) = \pi \left[ \delta(s(t) - s) - (\delta + \frac{\sigma}{\lambda})(p(t) - \bar{p}) \right], \tag{8.61}
\]

which is obtained by subtracting (8.60) from (8.59).

Now write (8.58) and (8.61) as the system

\[
\begin{pmatrix}
\dot{s}(t) \\
\dot{p}(t)
\end{pmatrix} = A \begin{pmatrix}
s(t) - \bar{s} \\
p(t) - \bar{p}
\end{pmatrix}, \tag{8.62}
\]

where

\[
A = \begin{pmatrix}
0 & 1/\lambda \\
\pi \delta & -\pi(\delta + \sigma/\lambda)
\end{pmatrix}.
\]

(8.62) is a system of two linear homogeneous differential equations. We know that the solutions to these systems take the form

\[
s(t) = \bar{s} + \alpha e^{\theta t}, \tag{8.63}
\]

\[
p(t) = \bar{p} + \beta e^{\theta t}. \tag{8.64}
\]

We will next substitute (8.63) and (8.64) into (8.62) and solve for the unknown coefficients, \( \alpha, \beta, \) and \( \theta \). First, taking time derivatives of (8.63) and (8.64) yields

\[
\dot{s} = \theta \alpha e^{\theta t}, \tag{8.65}
\]

\[
\dot{p} = \theta \beta e^{\theta t}. \tag{8.66}
\]
CHAPTER 8. THE MUNDELL-FLEMING MODEL

Substitution of (8.65) and (8.66) into (8.62) yields

\[(A - \theta I_2) \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = 0. \quad (8.67)\]

In order for (8.67) to have a solution other than the trivial one \((\alpha, \beta) = (0, 0)\), requires that

\[0 = |A - \theta I_2| = \theta^2 - Tr(A)\theta + |A|, \quad (8.68)\]

\[(146) \Rightarrow \quad \text{where } Tr(A) = -\pi(\delta + \sigma/\lambda) \text{ and } |A| = -\pi\delta/\lambda \text{ otherwise, } (A - \theta I_2)^{-1} \text{ exists which means that the unique solution is the trivial one, which isn’t very interesting. Imposing the restriction that (8.69) is true, we find that its roots are}

\[\theta_1 = \frac{1}{2}[Tr(A) - \sqrt{Tr^2(A) - 4|A|}] < 0, \quad (8.70)\]

\[\theta_2 = \frac{1}{2}[Tr(A) + \sqrt{Tr^2(A) - 4|A|}] > 0. \quad (8.71)\]

The general solution is

\[s(t) = \bar{s} + \alpha_1 e^{\theta_1 t} + \alpha_2 e^{\theta_2 t}, \quad (8.72)\]

\[p(t) = \bar{p} + \beta_1 e^{\theta_1 t} + \beta_2 e^{\theta_2 t}. \quad (8.73)\]

This solution is explosive, however, because of the eventual dominance of the positive root. We can view an explosive solution as a bubble, in which the exchange rate and the price level diverge from values of the economic fundamentals. While there are no restrictions within the model to rule out explosive solutions, we will simply assume that the economy follows the stable solution by setting \(\alpha_2 = \beta_2 = 0\), and study the solution with the stable root

\[\theta \equiv -\theta_1 \quad (8.74)\]

\[= \frac{1}{2}[\pi(\delta + \sigma/\lambda) + \sqrt{\pi^2(\delta + \sigma/\lambda)^2 + 4\pi\delta/\lambda}]. \quad (8.75)\]

Now, to find the stable solution, we solve (8.67) with the stable root

\[0 = (A - \theta_1 I_2) \begin{pmatrix} \alpha \\ \beta \end{pmatrix}\]

\[= \begin{pmatrix} -\theta_1 & 1/\lambda \\ \pi\delta & -\theta_1 - \pi(\delta + \sigma/\lambda) \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}. \quad (8.76)\]
When this is multiplied out, you get

\[ 0 = -\theta_1 \alpha + \beta / \lambda, \]  
\[ 0 = \pi \delta \alpha - [\theta_1 + \pi \left( \delta + \frac{\sigma}{\lambda} \right)] \beta. \]  

(8.77)  
(8.78)

It follows that

\[ \alpha = \beta / \theta_1 \lambda. \]  

(8.79)

Because \( \alpha \) is proportional to \( \beta \), we need to impose a normalization. Let this normalization be \( \beta = p_o - \bar{p} \) where \( p_o \equiv p(0) \). Then \( \alpha = (p_o - \bar{p}) / \theta_1 \lambda = -[p_o - \bar{p}] / \theta \lambda \), where \( \theta \equiv -\theta_1 \). Using these values of \( \alpha \) and \( \beta \) in (8.63) and (8.64), yields

\[ p(t) = \bar{p} + [p_o - \bar{p}] e^{-\theta t}, \]  
\[ s(t) = \bar{s} + [s_o - \bar{s}] e^{-\theta t}, \]  

(8.80)  
(8.81)

where \( (s_o - \bar{s}) = -[p_o - \bar{p}] / \theta \lambda \). This solution gives the time paths for the price level and the exchange rate.

To characterize the system and its response to monetary shocks, we will want to phase diagram the system. Going back to (8.58) and (8.61), we see that \( \dot{s}(t) = 0 \) if and only if \( p(t) = \bar{p} \), while \( \dot{p}(t) = 0 \) if and only if \( s(t) - \bar{s} = (1 + \sigma / \lambda \delta)(p(t) - \bar{p}) \). These points are plotted in Figure 8.10. The system displays a saddle path solution.
Figure 8.10: Phase diagram for the Dornbusch model.
Problems

1. (Static Mundell-Fleming with imperfect capital mobility). Let the trade balance be given by $\alpha(s + p^* - p) - \psi y$. A real depreciation raises exports and raises the trade balance whereas an increase in income leads to higher imports which lowers the trade balance. Let the capital account be given by $\theta(i - i^*)$, where $0 < \theta < \infty$ indexes the degree of capital mobility. We replace (8.3) with the external balance condition

$$\alpha(s + p^* - p) - \psi y + \theta(i - i^*) = 0,$$

that the balance of payments is 0. (We are ignoring the service account.) When capital is completely immobile, $\theta = 0$ and the balance of payments reduces to the trade balance. Under perfect capital mobility, $\theta = \infty$ implies $i = i^*$ which is (8.3).

(a) Call the external balance condition the FF curve. Draw the FF curve in $r, y$ space along with the LM and IS curves.

(b) Repeat the comparative statics experiments covered in this chapter using the modified external balance condition. Are any of the results sensitive to the degree of capital mobility? In particular, how do the results depend on the slope of the FF curve in relation to the LM curve?

2. How would the Mundell-Fleming model with perfect capital mobility explain the international co-movements of macroeconomic variables in Chapter 5?

3. Consider the Dornbusch model.

(a) What is the instantaneous effect on the exchange rate of a shock to aggregate demand? Why does an aggregate demand shock not produce overshooting?

(b) Suppose output can change in the short run by replacing the IS curve (8.7) with $y = \delta(s - p) + \gamma y - \sigma i + g$, replace the price adjustment rule (8.8) with $\ddot{p} = \pi(y - \bar{y})$, where long-run output is given by $\bar{y} = \delta(\bar{s} - \bar{p}) + \gamma \bar{y} - \sigma i^* + g$. Under what circumstances is the overshooting result (in response to a change in money) robust?