II. Endogenous Growth

Part II

• iii. LBD and spillover

Extended endogenous models–LBD and spillover

- Level of tech is determined by the capital-labor ratio:

\[ \bar{A} = A \left( \frac{K}{L} \right)^{\beta} \]

\[ Y = \bar{A} K^\alpha L^{1-\alpha} \]
\[ = AK^{a+\beta} L^{1-\alpha-\beta} \]

\[ y = Ak^{a+\beta} \]

- Utility function: \( \max \int_{0}^{\infty} e^{-\rho t} u(c_t) dt. \)

where \( u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma} \)
Growth rate of per capita consumption

Three cases:

1. $\alpha + \beta = 1$
   
   $g_c = \frac{f'(k) - \rho - n}{\sigma} = \frac{(\alpha + \beta) A k^{\alpha+\beta-1} - \rho - n}{\sigma}$
   
   $\Rightarrow g_c = \frac{(\alpha + \beta) A - \rho - n}{\sigma}$

2. $\alpha + \beta > 1$
   
   $g_c = \frac{(\alpha + \beta) A k^{\alpha+\beta-1} - \rho - n}{\sigma}$

   no steady state levels or growth rates
Growth rate of per capita consumption

Three cases:

3. $\alpha + \beta < 1$

$$g_c = \frac{(\alpha + \beta) A k^{\alpha + \beta - 1} - \rho - n}{\sigma}$$

$g_c = g_k = 0$ in the steady state since $MP_k$ declines as $k$ accumulates.
Learning by doing and knowledge spillover

- Eliminate DR so that we can have $g>0$
- Two assumptions:
  1. LBD: $K\uparrow$, firm learns simultaneously to produce more efficiently
     - Arrow (1962) inspired by empirical observation of positive effect of experience on productivity in airframe manufacturing and shipbuilding.
Learning by doing and knowledge spillover

2. Knowledge spillover: a piece of knowledge once discovered spillover across the whole economy
   - Knowledge has a non-rival nature; if one firm uses an idea, it doesn’t prevent others from using it; creation was a side product of I.
   - Romer (1986) enlarging the concept of $K$ and consider not only the accumulation of $K$ but also investment in knowledge induce external effect on other firm’s productivity
Learning by doing and knowledge spillover

- No effective patent system, knowledge ~ public goods
- All discoveries are unintended by-products of investment and that these discoveries immediately become common knowledge.
- This specification allows us to retain the framework of perfect competition; however, there is externality.
iii. AK + LBD (Romer, 1986)

\[
A = A \left( \frac{K}{L} \right)^{1-\alpha} \quad \Rightarrow \quad Y_j = \overline{A} K_j^\alpha L_j^{1-\alpha} = A \left( \frac{K}{L} \right)^{1-\alpha} K_j^\alpha L_j^{1-\alpha}
\]

\[
y_j = A \overline{k}^{1-\alpha} k_j^\alpha
\]

- \(k_j\): individual firm's level of \(K\) per worker
- \(\overline{k}\): economy-wide average level of \(K\) per worker

- Firm \(j\) faces DR to its own investment \(k_j\), but the production exhibits CRTS in \(k_j\) and \(\overline{k}\) taken together

- Production process generate knowledge externalities (Arrow, 1962): The higher the average level of \(k\) (capital intensive) in the economy, the greater the incentive of tech. spillovers that raises the MP\(_k\) throughout the economy.
With externality CE ≠ PO

\[ y_j = A\bar{k}^{1-\alpha}k_j^\alpha \]
\[ \frac{dy_j}{dk_j} = \alpha A \left( \frac{\bar{k}}{k_j} \right)^{1-\alpha} \]

in equilibrium,
since firms are symmetry,
\[ \Rightarrow k_j = \bar{k} \]
\[ \Rightarrow \frac{dy}{dk} = \alpha A = r \]

\[ g_c = \frac{\dot{c}}{c} = \frac{(r - \rho)}{\theta} \]

i. Market equilibrium: CE
\[ r = \alpha A \]
\[ \Rightarrow g_{c,CE} = (\alpha A - \rho)/\sigma \]

ii. PO: AK production
\[ r = A \]
\[ \Rightarrow g_{c,PO} = (A - \rho)/\sigma \]
With externality CE ≠ PO

\[ r = \rho + \sigma g_c \]

Social optimum

\[ (\alpha A - \rho)/\theta \]

\[ (A - \rho)/\theta \]
Important features

1. Market outcome is not optimal
   • Market interest rate $r = \alpha A$ is less than the social optimal $r^* = A$
   • Market equilibrium growth rate is less than the social optimal growth rate
   • Since firms do not internalize the LBD externality their investment produce for other firms.
   • Government’s subsidy raising the private $MP_k$ from $\alpha A$ to $A$ will induce firms to invest at the social optimal level
Important features

2. No convergence

- Two closed economies that are alike except for their initial \( k \) do not converge to identical per capita output levels. Only their growth rates converge.

- The early endogenous growth literature viewed this as a major advantage of the model, arguing that it could thus potentially explain why the convergence evidence appears to be so mixed.
Evidence from fast-growing East Asia

- AK – capital deepening can be an engine of sustained high growth rates
  - High rate of I in both physical K and education
- Is there ability to sustain high growth in favor of an AK-type LBD model?

Growth rates of real per capita GDP (1966-90)
CRTS or DRTS?

*Young (1992, 1994, 1995)*

- East Asian countries had experienced DR to $K$ as told in Solow growth model. No evidence of LBD externality.
- Empirical method: growth accounting
- Translog production function with quality adjusted $K$ and $L$ (education)
Young: 1. No extraordinary TFP

i. GDP per worker grows less rapidly than GDP per capita
   – High education attainment
   – Increasing rate of labor force participation

ii. Facts: Korea and Singapore, I/GDP: 30~40%;
   – High and increasing rates of investment.

iii. Productivity growth rates seem less impressive

East Asian countries have gains in TFP, but not super normal gains; Singapore: 0.2%
Young

2. Nothing of importance in the East Asian growth experience that cannot be accounted for by a model with CRTS in $K$, $H$ and raw labor taken jointly, and DR in each input taken individually.

3. The period of high growth—growth that has been achieved largely through sacrifice of current consumption and leisure—must eventually come to an end, just as the basic neoclassical model suggested.