III. Endogenous Growth

- The R&D Model
- The Government-Growth Model

(i). R&D-Endogenous innovation: P. Romer (1990)

- The empirical study of Young does not support the view that a country can sustain indefinite growth in per capita income through physical and human capital deepening alone.
- P. Romer provided an alternative endogenous growth model: invention is a purposeful economic activity that requires real resources.
- “Micro” side of new growth theory: explicitly modeling R&D process, one can gain important insights into the effects of both government policy and international integration on growth.
Why do firms innovate?

• R&D vs. “nonrival” ideas

• A unit of $K$ can only be used by one firm at a time; but through technology spillovers innovations may be used by more firms

• If firm has an incentive to innovate, there must exist some type of institutional mechanism that allow firms to appropriate rents from his invention

• Romer assumes that inventors can obtain patent licenses on the “blueprint” for their inventions
How will innovation affect production

- Channel I: Inventions
  - Variety/quality of consumer products
- Channel II: production efficiency
  
  R&D $\rightarrow$ new method for more efficient production

Focusing on channel II, Romer assumes

  i. R&D $\rightarrow$ new intermediate goods (capital)
     $\rightarrow$ enhance labor productivity
  
  ii. One homogenous consumption good
The Arguments

• Technological change:
  – Improvement in the instruction for mixing together raw material
  – lies at the heart of economic growth

• Technological changes arise in large part because of intentional action taken by people who respond to market incentives

• Instructions for working with raw materials are inherently different from other economic goods. Once the cost of creating a new set of instruction has been incurred, the instruction can be used over and over again at no additional cost.
Stages of production

• Research to develop design or technology
• Produce technology as capital goods
• Using technology (capital goods) to produce final product
• Notations:
  L – labor (fixed)
  H – level of human capital (fixed)
  K – stock of physical capital
  A – technology

\{ \text{exogenous} \}
Three sectors:
1. final consumer goods

- CE: free entry $\rightarrow$ no excess profit
- Final products are never obsolete
- New and old products are perfect substitute
- Stock of human $K$ can be divided into two sectors:

$$H \quad \begin{array}{c} \quad \rightarrow \quad \rightarrow \end{array} \quad H_Y: \text{produce final product}$$

$$H_A: \text{produce technology}$$

$$Y(H_Y, L, X)$$

$$= H_Y^\alpha L^\beta \int_0^A X_i^{1-\alpha-\beta} \, di$$

$x_i$: different type of capital goods; computers, machines...

$A$: range of capital goods invented
2. R&D sector: blueprint for new K

- Production of new design
  \[ \dot{A} = \delta H_A A \Rightarrow \frac{\dot{A}}{A} = \delta H_A \]

- Where \( \delta \) is a productivity parameter.
- The growth in the new product is the function of human capital and Technology level. All researcher can take advantage of \( A \).
- Assumptions:
  - The larger total stock of designs and knowledge is, the higher the productivity of an engineer working in the research sector will be.
  - Increasing human capital to research leads to a higher rate of production of new designs.
3. Intermediate goods sector

- Each $x_j$ is produced by a monopolist who is the only owner of the blueprint for this particular good.

- The monopolistic producer of good $j$ takes $\eta x$ unit of raw capital and converts them into $x$ unit of specialized capital good, using his blueprint.

<table>
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<tr>
<th>R&amp;D ($\ell$)</th>
<th>Design</th>
<th>Intermediate goods: new K</th>
<th>Monopoly supplier</th>
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how to combine $Y$ to produce new K
Final good sector: Demand for new capital goods

- Final goods sector’s demand for new $K$ depends on its price $p_j$. The quantity of $x_j$ is chosen to max profits:

$$\pi = \max_{x_i, H_Y} \left\{ \int_0^A \left[ H_Y^\alpha L^\beta x_i^{1-\alpha-\beta} - p_i x_i \right] di - w_H H_Y \right\}$$

$$\frac{\partial \pi}{\partial x_j} = (1 - \alpha - \beta) H_Y^\alpha L^\beta x_j^{-\alpha-\beta} - p_j = 0$$

$$\Rightarrow x_j = (1 - \alpha - \beta)^{\frac{1}{\alpha+\beta}} H_Y^{\frac{\alpha}{\alpha+\beta}} L^{\frac{\beta}{\alpha+\beta}} p_j^{-\frac{1}{\alpha+\beta}}$$

Each intermediate goods producer faces a constant-price-elasticity demand curve, so that 1% rise in $p_j$ leads to a $1/(\alpha+\beta)$ % fall in demand.
Demand for new capital goods

- Inverse demand $P(x)$:

$$p_j = (1 - \alpha - \beta) H_Y^\alpha L^\beta x_j^{-\alpha - \beta}$$

- It is downward sloping
- It is identical for each specialized capital good $j$
- Doubling the $H_Y$ and $L$ will double the demand for each specialized capital good $j$. 
Intermediate good sector

• Each monopolist faces the demand curve $P_j(x_j)$ and chooses the current quantity of $x$ to max her profit.

• Since the cost of raw capital equals to interest rate $r$, the interest rate acts as monopolist’s MC. So the profit is:

$$\pi_j = \max_x P_j(x_j) x_j - rK_j$$

$$= \max_x P_j(x_j) x_j - r\eta x_j$$
Intermediate good sector

- Each monopolist faces identical demand and has identical MC, \( r \). Hence, prices and quantities chosen by different monopolists are the same:
  - \( x_j = x \) for any \( j \)
  - \( P_j = P \) for any \( j \)
  - We no longer need the subscript \( j \), so we can drop it

\[
\max_x P(x)x - r\eta x
\]
Monopolistic pricing of new intermediate goods

- Intermediate goods sector:
  current profit = revenue - variable cost

\[
\max_x \left[ (1 - \alpha - \beta)H^\alpha L^\beta x^{1-\alpha-\beta} - r\eta x \right]
\]

\[
\frac{\partial \pi}{\partial x} = (1 - \alpha - \beta)^2 H^\alpha L^\beta x^{-\alpha-\beta} - r\eta = 0
\]

\[
\Rightarrow P = \frac{r\eta}{(1 - \alpha - \beta)} = MC(1 + \text{markup})
\]

\[
= (r\eta) \left( 1 + \frac{1}{\varepsilon_d - 1} \right); \varepsilon_d = \left| \frac{-1}{\alpha + \beta} \right|
\]

Monopoly Price is a simple mark-up over marginal cost, where the mark-up is determined by the elasticity of demand.
Note: A mark-up monopolistic pricing

$$\max_x P(x)x - r\eta x$$

$$MR = P(x) + xP'(x) = P\left(1 + \frac{xP'}{P}\right) = r\eta = MC$$

$$P = \frac{r\eta}{1 - \frac{1}{\varepsilon_d}}; \quad \frac{1}{\varepsilon_d} = -\frac{x}{P \frac{dP}{dx}} = \frac{1}{x} \left( -\alpha - \beta \right) \frac{P}{x} = \alpha + \beta$$

∴ $$P = (1 - \alpha - \beta) Y^\alpha L^\beta x^{-\alpha - \beta}$$

Monopolistic profit:

$$\pi = \max_x P(x)x - r\eta x$$

$$\Rightarrow \pi = (\alpha + \beta)Px$$
Production function in terms of $K$

- When quantity of every good demanded is the same and equal to $x$, then
  \[ Y = H_Y^\alpha L^\beta \int_0^A x^{1-\alpha-\beta} \, dx = H_Y^\alpha L^\beta Ax^{1-\alpha-\beta} \]

- Capital stock is simply: $K = \eta x A$
  \[ K = \eta \int_0^A x_i \, di = \eta Ax \]

- In terms of capital stock, the production function is the usual one:
  \[ Y = H_Y^\alpha L^\beta A^{\alpha+\beta} (K/\eta)^{1-\alpha-\beta} \]
  \[ = (AH_Y)^\alpha (AL)^\beta (K/\eta)^{1-\alpha-\beta} \]
Pricing of Patent

• Every time a new idea is produced, it is patented by the inventor.
• Assume that everyone can buy a patent for and start earning monopoly profits from them on.
• Patents are sold at an auction to the highest bidder. The highest bid must be exactly equal to the present value of all future monopoly profits generated by the patent.
• In the future, the same blueprint is going to be used by an increasing number of production workers. This is because the demand for \( x_j \) will grow in proportion to \( L \). That is, \( x_j \) will grow at rate \( n \) (same as \( L \)), and so must monopoly profit from selling \( x_j \).
Pricing of blueprints ($P_A$)

- Cost of a new investment $P_A$ must be equal to the present value of the net revenue that a monopolist can extract.
  - The bidding for the patent continues until
  - Value of blueprints = entire PV of the profit stream,
    $$P_A = V_A$$
    $$P_A(t) = \int_t^\infty e^{-rs} e^{ns} \pi(s) ds$$

- Suppose $n=0$, thus $\pi$ is constant,
  - $$V_A = \pi / r \Rightarrow \pi = rP_A$$

the instantaneous excess of revenue over variable cost must be just sufficient to cover the interest rate on initial investment in design.
Anyone engaged in research can freely take advantage of the entire stock of designs in doing research to produce new design, it follows that $P_A$ and $w_H$ are related by: $w_H = P_A (\delta A)$

\[
H_Y = \frac{\alpha r}{\delta (\alpha + \beta)(1 - \alpha - \beta)}
\]
Balanced growth

\[ K = \eta \int_0^A x_i \, di = \eta Ax \Rightarrow \frac{\dot{K}}{K} = \frac{\dot{A}}{A} \]

With Investment:

\[ \dot{K} = Y - c \Rightarrow \frac{\dot{K}}{Y} = 1 - \frac{c}{Y} \Rightarrow \frac{\dot{K}}{K} \frac{K}{Y} = 1 - \frac{c}{Y} \]

\[ \Rightarrow g_K \frac{K}{Y} = 1 - \frac{c}{Y} \]

\[ \frac{Y}{K} = \left( \frac{A}{K} H_Y \right)^\alpha \left( \frac{A}{K} L \right)^\beta (1/\eta)^{1-\alpha-\beta} \]

\[ g = \frac{\dot{c}}{c} = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{A}}{A} = \delta H_A \]
The endogenous growth rate – supply side

Since

\[ H_Y = \frac{\alpha r}{\delta (\alpha + \beta)(1 - \alpha - \beta)} \]

\[ H = H_A + H_Y \]

\[ \Rightarrow g = \delta H - \frac{\alpha}{(1 - \alpha - \beta)(\alpha + \beta)} r = \delta H - \Gamma r \]

where \( \Gamma \) is a constant that depends on the technology parameter \( \alpha \) and \( \beta \)
The optimal growth rate

Lifetime UTILITY: \( U(c) = \int_0^\infty U(c) e^{-\rho t} dt \) where \( U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma} \)

Euler equation of consumption:

\[
\frac{\dot{c}}{c} = \left( \frac{r - \rho}{\sigma} \right) = g \Rightarrow r = \rho + \sigma g
\]

- Growth does not depend on labor.
- It depends on the taste or preference parameter \( \rho \), elasticity of intertemporal substitution \( \sigma^{-1} \), and the tech parameters: the level of human capital \( H \), \( \delta \), and \( \Gamma \).
• If $H$ is too low ($H < \bar{H}$), the non-negative constraint on $H_A$ is binding and growth does not take place. If the total level of human capital is too small, stagnation may arise. This result offers one possible explanation for why there exist variations in growth rates among countries.

\[
g = \frac{\delta H - \Gamma \rho}{\sigma \Gamma + 1}
\]

\[
H = \frac{\Gamma \rho}{\delta}
\]
Two sources of inefficiency in CE

1. R&D sector → LBD externality
R&D firms do not take into account their invention will lower the amount of labor required to create future inventions

2. Monopolistic market structure: firms producing intermediate capital goods tend to produce less than the socially efficient quantity of K.
Thus, socially optimal growth rate is higher than the CE growth rate.
• Welfare-enhancing government intervention and international integration have positive effects on growth
Recap: Solow and Ramsey

- No autonomous engine of growth: in the absence of exogenous technical progress, growth dies off in the long-run.
  - No theory of determinants of long-run growth
  - No theory of determinants of long-run cross-country differences in growth rates
  - Policies do not affect long-run growth
Recap: Making Technology endogenous

- Endogenous R&D
  - Why not add a R&D production function
  - Expanding variety models
    - Romer (1990), Grossman and Helpman (1991)
    - New goods get introduced but old ones are never retired.
Endogenous R&D

• In the U.S. there has been a rise in the skill premium at the same time as an expansion in the number of skilled workers.

• The larger number of skilled workers increased the size of the market and made it more profitable to produce innovations that enhance the productivity of skilled workers.

AK model: Redefine the K

Introduce Externalities

- Romer (1986)
- \( Y = F(K, N, \underline{K}) \)
- \( \underline{K} \) – economy-wide capital stock
- Empirical evidence: no externalities in most industries (Burnside (1996))
Externalities

• Investment in knowledge has a "natural externality" -- that is, knowledge can't be perfectly patented or kept secret.

• Once you know that something can be done, you can start trying to duplicate it. And new knowledge has a positive effect on the production possibilities of other firms.
Examples: externality

- IBM developers took their competition's product apart and counted the number of parts, finding that the other printer had fewer parts. By working towards fewer parts, they weren't exactly reverse-engineering the competition's printer (although that must also have happened), they were also making their production process more efficient.

- As a side benefit, the knowledge that fewer parts to put together meant cheaper production, coupled with a highly profitable example, led other companies to streamline production, even if they weren't in the same fields.
Examples: externality

The Internet is a great example. It was developed by the US military to have no central controls, just in case the country needed to survive a nuclear attack. What it's become is a metaphor for the global village -- no center, all connected.
But endogenous growth doesn't just happen

- There are a few preconditions.
  - As Romer writes in his 1990 paper, "Endogenous Technological Change," the model of endogenous growth has 4 basic inputs:
- Capital & Labor
- Human capital -- activities such as formal education and on-the-job training. This is person-specific; if the person who knows how to multiply dies, that skill is lost from the pool of human capital
- An index of the level of the technology
Romer’s R&D model: human capital

"what is important for growth is integration not into an economy with a large number of people but rather into one with a large amount of human capital" (Romer, 1990, S98).
Growth-promoting economic policies

1. encourage investment in new research, as opposed to encouraging investment in physical capital accumulation.
   or, if 1 is not possible, at least:

2. subsidize the accumulation of total human capital.
Two interesting implications

1. That open trade may be supportive of growth and technological development.
   - The example he gives is a study on US counties in the early 19th century. The ones which were close to navigable waterways had higher rates of patenting than those which were inland; as water transportation was introduced, the rate of patenting went up. (Of course this may have something to do with the fact that opening up the areas made them more attractive for creative people to work there, but this is also a metaphor for creative people in the global economy.)
Two interesting implications

2. That therefore, "a less developed economy with a very large population can still benefit from economic integration with the rest of the world" (p. s99). However, the "economy with the larger total stock of human capital will experience faster growth."
AK model: Redefine the K

II. Incorporate Human Capital

- \( Y = F(K, N, H) \)

- Implies that worker productivity can grow without bound in the absence of technical progress

AK model can be thought as a reduced-form representation. An example with physical and human capital.

\[ Y = A K^\alpha H^{1-\alpha} \]
Long-run growth effect: AK

- Policies can now affect long-run growth
- A permanent proportional tax on returns to capital
  - Firms pay a rental rate: $R_t = A$
  - Consumers gross return to savings: $R_t(1-\tau)$
  - $r_t = A(1-\tau) - \delta_t$
  - Growth rate: $[A(1-\tau) - \delta \rho]/\theta$
AK model: Redefine the K

III. Government and growth
- AK as a reduced form
- Assume \( n=0 \)
- Government taxes agents or firms and uses the proceeds to provide free public services to producers.
- Government spend the tax receipts in public goods used by all firms simultaneously and with no congestion effect.
Government and Growth

- Production function: $Y_i = AL_i^{1-\alpha} K_i^\alpha G^{1-\alpha}$
  - CRS: $(L_i, K_i)$; IRS: overall
  - CRS: reproducible inputs $(K, G)$

If $G$ and $K$ grow at a constant rate, the return to capital does not fall over time.

Note: if the exponent of $G$ were smaller than $1-\alpha$, then we have diminishing returns to reproducible inputs, and no sustained growth.
Government and Growth

- Balance budget: \( G = \tau Y \); \( \tau \) is the tax rate which is assumed to be levied on the value of production of each firm. Thus,

\[
G = \left( \tau AL \right)^{1/\alpha} k
\]

The firms’ after tax profit is:

\[
\Pi_i = L_i \left[ (1 - \tau) Ak_i^\alpha G^{1-\alpha} - w - (r + \delta) k_i \right]
\]

Profit max implies that:

\[
r + \delta = \alpha (1 - \tau) A \left( G/k_i \right)^{1-\alpha}
\]
Government and Growth

- Hence, substituting $G$ by its expression, we obtain:

$$r + \delta = \alpha (1 - \tau) A^{1/\alpha} (\tau L)^{(1-\alpha)/\alpha}$$

- from the standard Euler equation, we have then

$$\frac{\dot{c}}{c} = \frac{1}{\sigma} \left[ \alpha (1 - \tau) A^{1/\alpha} (\tau L)^{(1-\alpha)/\alpha} - \delta - \rho \right]$$

$$\dot{k} = A^{1/\alpha} (\tau L)^{(1-\alpha)/\alpha} k - \delta k - c$$

- the equilibrium has, as usual, constant growth, given by:

$$g = \frac{1}{\omega} \left[ \alpha (1 - \tau) A^{1/\alpha} (\tau L)^{(1-\alpha)/\alpha} - \delta - \rho \right]$$
Government expenditure \((\tau = G/Y)\) has two opposite effects on growth:

1. \((1-\tau)\): growth-depressing distortionary effect; negative effect of taxation on the \(\text{MP}_k\);
2. \((\tau^{(1-\alpha)/\alpha})\): growth–enhancing effect of public services; positive effect of public good on the \(\text{MP}_k\)

\[
g = \left(\frac{1}{\sigma}\right) \left[ \alpha (1-\tau) A^{1/\alpha} \left(\tau L \right)^{(1-\alpha)/\alpha} - \delta - \rho \right]
\]
Inverse U-shaped relationship between growth and $\tau$

\[ g = \left(1/\sigma\right)\left[ \alpha(1 - \tau) A^{1/\alpha} (\tau L)^{(1-\alpha)/\alpha} - \delta - \rho \right] \]

\[ \frac{\partial g}{\partial \tau} = 0 \Rightarrow (1 - \alpha - \tau) A^{1/\alpha} \tau^{[\frac{(1-\alpha)/\alpha]}-1} L^{(1-\alpha)/\alpha} = 0 \]

\[ \Rightarrow \tau^* = 1 - \alpha \]

- The maximum is achieved in correspondence of the condition $\tau = G/Y = (1-\alpha)$.
- Interpretation: equating the marginal cost of capital to marginal benefit
Inverse U-shaped relationship between growth and $\tau$
Comparing with AK

- Dynamics are the same as those of the AK model: no transitional dynamics

\[ \frac{\dot{c}}{c} = \left(1/\sigma\right) \left[ \alpha (1-\tau) A^{1/\alpha} (\tau L)^{(1-\alpha)/\alpha} - \delta - \rho \right] \]

\[ \frac{k}{k} = A^{1/\alpha} (\tau L)^{(1-\alpha)/\alpha} - \delta - \frac{c}{k} \quad \rightarrow \quad \frac{c}{k} \text{ is a constant} \]

- Two differences:
  1. Scale effects
  2. Pareto non-optimality